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PROPAGATION OF ELASTIC VIBRATIONS IN SNOW

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At rather low temperatures snow is characterized by three velocities of sound, two of which are related to propagation of longitudinal and transverse waves through a solid skeleton formed by ice crystals and the third of which is related to propagation of longitudinal waves through air in snow pores. The main laws governing propagation and absorption of these waves are determined. Analytical formulas that express the dependence of the coefficient of attenuation of waves on frequency are obtained.

Snow is a porous medium that consists of a solid skeleton formed by ice crystals tightly pressed to each other and air that fills the space between these crystals. Air can freely move through the pores, both in and out.

Near a zero temperature, ice crystals are covered by a water film, which considerably decreases the rigidity of the solid skeleton, and snow loses elastic properties. As temperature decreases, sintering of crystal grains occurs at the places of contact; the solid skeleton becomes rigid and acquires the ability to resist deformation under the action of the applied load. In the case of small short loads and deformations under which the bridges between crystal grains are not broken, the solid skeleton becomes elastic and longitudinal and transverse waves can propagate through it. Moreover, longitudinal waves can propagate through the air confined in the snow pores. Thus, snow, in contact with ordinary solid bodies, possesses three velocities of sound. Each of these velocities is in a certain manner related to the main parameters that characterize the elastic properties of snow and its structure. Investigation of these relations is an important problem of the physics of snow. However, up to now there have been no theoretical and experimental studies of the laws governing propagation and absorption of different types of waves in snow. These studies are of great interest and allow one to extend the possibilities of investigation of physicomechanical properties of snow by acoustic methods.

This work is devoted to investigation of the laws governing propagation of different types of harmonic waves in an unbounded volume of snow and determination of relations between the parameters of these waves and elastic characteristics of snow.

Equations of Snow Motion with Account for Friction Forces between the Solid and Gaseous Phases. In what follows, we consider only dry snow, i.e., snow at temperatures below $-(2–3)^{\circ}$C, where ice crystals form a solid skeleton that possesses elasticity and free spaces between ice crystals are filled by air. In this case, snow, as a two-component medium, is absolutely similar to water-saturated soil: ice crystals play the role of sand grains that form the soil skeleton; the role of liquid that fills the pores between sand grains is played by the air confined in snow pores. Consequently, the equations describing the snow motion will, in many respects, coincide with the equations describing the motion of water-saturated soil, which were first obtained by Ya. I. Frenkel' [1].

As is shown in [1], motion of the solid skeleton and air in snow pores, with account for friction forces arising between them according to the Darcy law, is described by the system of equations

$$\rho_2 \frac{\partial \mathbf{v}_2}{\partial t} = K_2 \nabla\varphi - \frac{\eta_2 f}{k} (\mathbf{v}_2 - \mathbf{v}_1) , \tag{1}$$

$$\rho_1 (1 - f) \frac{\partial \mathbf{v}_1}{\partial t} = (\lambda + 2\mu) \nabla\theta + \mu\Delta\mathbf{u} + \left(1 - f - \frac{K}{K_1}\right) K_2 \nabla\varphi + \frac{\eta_2 f}{k} (\mathbf{v}_2 - \mathbf{v}_1) , \tag{2}$$

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$$\frac{\partial}{\partial t}(\rho_2 f) + \mathrm{div}\,(\rho_2 f \mathbf{v}_2) = 0\,, \tag{3}$$

$$\rho_2 = \rho_{2,0}\,(1 - \varphi)\,, \tag{4}$$

$$\Delta f = (1 - f)\left(\theta - \frac{K_2}{K_1}\varphi\right). \tag{5}$$

We linearize this system, assuming $\mathbf{v}_1$, $\mathbf{v}_2$, $\mathbf{u}$, θ, φ, and $\Delta\varphi$ to be small quantities of the first order of smallness, and present f and ρ_2 in the form $f = f_0 + f'$, $\rho_2 = \rho_{2,0} + \rho_2'$, where f_0 is the value of f that corresponds to the absence of deformation; f' and ρ_2' are small additions. Then

$$\rho_1\,(1 - f_0)\,\frac{\partial \mathbf{v}_1}{\partial t} = (\lambda + 2\mu)\,\nabla\theta + \mu\Delta\mathbf{u} + \left(1 - f_0 - \frac{K}{K_1}\right)K_2\nabla\varphi + \frac{\eta_2 f_0}{k}\,(\mathbf{v}_2 - \mathbf{v}_1)\,, \tag{6}$$

$$\rho_{2,0}\,\frac{\partial \mathbf{v}_2}{\partial t} = K_2\nabla\varphi - \frac{\eta_2 f_0}{k}\,(\mathbf{v}_2 - \mathbf{v}_1)\,, \tag{7}$$

$$f_0\,\frac{\partial \rho'}{\partial t} + \rho_{2,0}\,\frac{\partial\,(\Delta f)}{\partial t} + \rho_{2,0} f_0\,\mathrm{div}\,\mathbf{v}_2 = 0\,, \tag{8}$$

$$\Delta f = (1 - f_0)\left(\theta - \frac{K_2}{K_1}\varphi\right) = 0\,. \tag{9}$$

The coefficients K, μ, and λ in these equations refer to the solid skeleton of snow; they are expressed in terms of the Young modulus E and the Poisson coefficient σ by the formulas [2]

$$\mu = \frac{E}{2\,(1+\sigma)}\,,\quad K = \frac{E}{3\,(1 - 2\sigma)}\,,\quad \lambda = \frac{E\sigma}{(1+\sigma)\,(1-2\sigma)}\,.$$

In the range of naturally occurring snow densities 100–500 kg/m^3, E changes within three orders from 10^6 to 10^9 Pa. The Young modulus E weakly depends on temperature: a decrease in temperature from -3°C to -20°C leads to only a 15–20% increase of it [3]. The Poisson coefficient σ can be taken equal to 0.3. The moduli of three-dimensional compression of ice and air are $K_1 = 2\cdot 10^9$ Pa and $K_2 = 1.4\cdot 10^5$ Pa, respectively.

Unfortunately, we do not have any data on the coefficients of permeability of different types of snow. However, in order to estimate the value of this coefficient we can use one of available semi-empirical formulas, e.g., the Kozeni formula [4], which in our notation is written as

$$k = \frac{f^3\delta^2}{150\,(1-f)^2}\,.$$

For snow, the coefficient of porosity f changes from 0.2 to 0.7 and δ from 0.05 to 0.5 cm. In this case, $10^{-7} < k < 6\cdot 10^{-3}$ cm^2.

Propagation of Longitudinal and Transverse Vibrations in Snow. In order to obtain the equation describing propagation of longitudinal waves in snow relating only to variation of a volume element, we must apply the div operation to Eqs. (6) and (7). It follows from Eq. (8) that

$$\rho_{2,0}\frac{\partial}{\partial t}\,\mathrm{div}\,\mathbf{v}_2 = K_2\,\mathrm{div}\,\nabla\varphi - \frac{\eta_2 f_0}{k}\,(\mathrm{div}\,\mathbf{v}_2 - \mathrm{div}\,\mathbf{v}_1)\,. \tag{10}$$

Substituting the value of Δf from (9) in (8) and allowing for the equality

$$\frac{1}{\rho_{2,0}}\frac{\partial\rho'}{\partial t} = \frac{\partial}{\partial t}\left(\frac{\rho'}{\rho_{2,0}}\right) = -\frac{\partial\varphi}{\partial t}\,,$$

we obtain

$$\mathrm{div}\,\mathbf{v}_2 = -\left(\frac{1}{f_0} - 1\right)\frac{\partial\theta}{\partial t} + \left[1 + \left(\frac{1}{f_0} - 1\right)\frac{K_2}{K_1}\right]\frac{\partial\varphi}{\partial t}\,, \tag{11}$$

$$\mathrm{div}\,\mathbf{v} = \mathrm{div}\,\frac{\partial\mathbf{u}}{\partial t} = \frac{\partial}{\partial t}\,\mathrm{div}\,\mathbf{u} = \frac{\partial\theta}{\partial t}\,. \tag{12}$$

Substituting (11) and (12) in Eq. (10) and taking into account that $K_2/K_1 << 1$, we have

$$(1 - f_0)\frac{\partial^2\theta}{\partial t^2} - f_0\frac{\partial^2\varphi}{\partial t^2} + f_0 c_2^2\Delta\varphi + \frac{\nu_2 f_0}{k}\left(\frac{\partial\theta}{\partial t} - \frac{\partial\varphi}{\partial t}\right) = 0\,, \tag{13}$$

where $\nu_2 = \eta_2/\rho_{2,0}$ is the kinematic coefficient of air viscosity and $c_2 = \sqrt{K_2/\rho_{2,0}}$ is the adiabatic velocity of sound in air.

In the same manner, from Eq. (6) we have

$$\frac{\partial^2\theta}{\partial t^2} - c_\ell^2\Delta\theta - \varepsilon\left(1 - f_0 - \frac{K}{K_1}\right)c_2^2\Delta\varphi + \frac{\varepsilon\nu_2 f_0}{k}\left(\frac{\partial\theta}{\partial t} - \frac{\partial\varphi}{\partial t}\right) = 0\,, \tag{14}$$

where $\varepsilon = (\rho_2/\rho) \sim 10^{-3}$–$10^{-2}$, $c_l = \sqrt{(\lambda + 2\mu)/\rho}$, and $\rho = \rho_1(1 - f_0)$.

Thus, the system of equations (13) and (14) describes propagation of longitudinal waves in snow.

In order to obtain equations describing propagation of transverse waves we apply the rot operation to Eqs. (6) and (7). Introducing the notation $\boldsymbol{\omega}_1 = \frac{1}{2}$ rot $\mathbf{v}_1$, $\boldsymbol{\omega}_2 = \frac{1}{2}$ rot $\mathbf{v}_2$ and allowing for the identities rot $\nabla\varphi = 0$, rot $\nabla\mathbf{u} = \nabla$ rot $\mathbf{u}$, we obtain

$$\frac{\partial^2\boldsymbol{\omega}_1}{\partial t^2} = c_\mathrm{t}^2\Delta\boldsymbol{\omega}_1 + \frac{f_0\eta_2}{k\rho}\left(\frac{\partial\boldsymbol{\omega}_2}{\partial t} - \frac{\partial\boldsymbol{\omega}_1}{\partial t}\right) = 0\,, \tag{15}$$

$$\frac{\partial\boldsymbol{\omega}_2}{\partial t} = -\frac{\nu_2}{k}\,(\boldsymbol{\omega}_2 - \boldsymbol{\omega}_1)\,, \tag{16}$$

where $c_\mathrm{t} = \sqrt{\mu/\rho}$.

Propagation of Longitudinal Harmonic Vibrations in Snow. We seek the solution of Eqs. (13) and (14) in the form of travelling sinusoidal waves

$$\theta = \theta_0\exp\,(i\,(qx - \omega t))\,, \quad \varphi = \varphi_0\exp\,(i\,(qx - \omega t))\,. \tag{17}$$

Here θ_0 and φ_0 are the amplitudes of vibrations which depend only on the space coordinates.

Substituting (17) in (13) and (14), we obtain the linear system of equations relative to θ_0 and φ_0. The condition of compatibility of this system leads to the dispersion equation

$$c_2^2 c_\ell^2 \xi^2 - \left[c_2^2 + c_\ell^2 + \left(\frac{1}{f_0} - 1 \right) \varepsilon' c_2^2 + (c_\ell^2 + \varepsilon c_2^2 + \varepsilon' c_2^2)\, \chi i \right] \xi^2 + 1 + i\chi\,(1 + \varepsilon) = 0\,,$$

where $\xi = \frac{q}{\omega}$, $\chi = \frac{\nu_2}{k\omega}$, and $\varepsilon' = \left(1 - f_0 - \frac{K}{K_1}\right)\frac{\rho_{2,0}}{\rho}$.

In this equation, the terms involving the factors ε and ε' are negligibly small; therefore we can write

$$c_2^2 c_\ell^2 \xi^4 - (c_2^2 + c_\ell^2 + i\chi c_\ell^2)\, \xi^2 + 1 + i\chi = 0\,. \tag{18}$$

Solution of (18) relative to ξ^2 gives the following equation:

$$\xi_{1,2}^2 = \frac{1}{2c_2^2}\left[(m + 1 + i\chi) \pm \sqrt{(m + 1 + i\chi)^2 - 4m\,(1 + i\chi)} \right], \tag{19}$$

where $m = c_l^2/c_2^2$. With account for the fact that the radicand is $(m + 1 + i\chi)^2 - 4m(1 + i\chi) = (m - 1 - i\chi)^2$, (19) takes on the form

$$\xi_{1,2}^2 = \frac{(m + 1 + i\chi) \pm (m - 1 - i\chi)^2}{2c_2^2}\,.$$

Hence we have

$$\xi_1 = \frac{1}{c_\ell}\,, \quad \xi_2 = \frac{1}{c_2}\sqrt{1 + i\chi}\,.$$

The wave vector $q_1 = \omega\xi_1 = \frac{\omega}{c_l}$ refers to the solid skeleton and the wave vector $q_2 = \omega\xi_2 = \frac{\omega}{c_2}\sqrt{1 + i\chi}$ refers to the air confined in the snow pores.

Thus, the first phase velocity of sound in snow is c_l and does not depend on the frequency of vibrations; it coincides with the velocity of sound propagation in the solid skeleton of snow.

We present the wave vector q_2 as

$$q_2 = q_2' + iq_2''\,,$$

where

$$q_2' = \frac{\omega}{\sqrt{2}\,c_2}\sqrt{\sqrt{1 + \chi^2} + 1}\,; \quad q_2'' = \frac{\omega}{\sqrt{2}\,c_2}\sqrt{\sqrt{1 + \chi^2} - 1}\,.$$

Here $q_2'' = \gamma_2$ is the coefficient of absorption of sound in air confined in the pores, and the real part of the wave number q_2' is related to the phase velocity v_{p} by the formula

$$v_{\mathrm{p}} = \frac{\omega}{q_2'} = \frac{\sqrt{2}\,c_2}{\sqrt{\sqrt{1 + \chi^2} + 1}}\,. \tag{20}$$

It is seen from expression (20) that the phase velocity of sound in the snow pores depends on frequency ω in terms of the parameter χ. This means that air inclusions in snow form a dispersing medium: the velocity of propa-

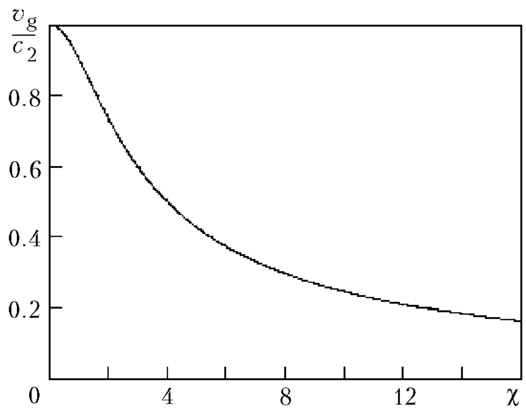

Fig. 1. Dependence of the dimensionless pore velocity of sound on the parameter $\chi = \nu^2/k\omega$.

gation of a monochromatic wave depends on frequency. The velocity of sound propagation in such medium is equal to the group velocity $v_g = \dfrac{d\omega}{dq_2'}$.

Differentiating q_2' with respect to ω, we obtain

$$\frac{dq_2'}{d\omega} = \frac{1}{\sqrt{2}\, c_2}\left[\sqrt{\sqrt{1+\chi^2}+1} - \frac{\chi^2}{2\sqrt{1+\chi^2}\,\sqrt{\sqrt{1+\chi^2}+1}}\right].$$

Hence, for the group velocity we have the formula

$$v_g = 2\sqrt{2}\, c_2 \frac{(1+\chi^2)^{1/4}}{\left(1+\sqrt{1+\chi^2}\right)^{3/2}} . \tag{21}$$

Thus, the pore velocity of sound in snow depends only on the dimensionless parameter χ and does not depend on the type of snow and its density. In this sense, the process of propagation of sound through air inclusions of snow possesses self-similarity.

Figure 1 presents the dependence of the pore velocity of sound, which is equal to the group velocity v_g, on χ. An important special feature of this curve is the fact that it does not depend on the type of snow and is universal.

Propagation of elastic vibrations through air inclusions of snow is possible provided only that the distance at which the wave amplitude decreases *e*-fold exceeds the wave length $\lambda_2 = \dfrac{v_g}{2\pi\omega}$, i.e., the condition $\dfrac{2\pi v_g}{\omega} < \dfrac{1}{\gamma_2}$ must hold; this condition can be written as

$$\frac{\left(1+\sqrt{1+\chi^2}\right)^2}{\chi\,(1+\chi^2)^{1/4}} > 4\pi .$$

Hence we obtain

$$\chi \le 0.33 . \tag{22}$$

On the other hand, the frequency ω cannot be as large as is wished. Its value is limited from above by the requirement that the wave length $\lambda_2 = \dfrac{2\pi v_g}{\omega} \approx \dfrac{2\pi c_2}{\omega}$ greatly exceed the linear dimensions of the pores δ, i.e., the condition

$$\omega << \frac{2\pi c_2}{\delta} \tag{23}$$

must hold.

Physically, conditions (22) and (23) indicate that propagation of harmonic waves through air inclusions of snow is possible only under the condition that the snow density be rather small and the frequencies of vibrations not too large. The estimates show that at $k = 10^{-3}$–10^{-6} cm^2 the operating range of frequencies at which acoustic measurements in snow can be made is 10^2–10^5 Hz.

Transverse Harmonic Vibrations in Snow. Considering the solutions of Eqs. (15) and (16) in the form of traveling harmonic vibrations, we come to the system of equations

$$\left(qc_t^2 - \omega^2 + i\,\frac{\nu_2 f_0 \omega}{k\rho}\,\varepsilon\right)\boldsymbol{\omega}_1 - i\,\frac{\nu_2 f_0 \varepsilon\omega}{k}\,\boldsymbol{\omega}_2 = 0\,,$$

$$\frac{\nu_2}{k}\,\boldsymbol{\omega}_2 - \left(\frac{\nu_2}{k} + i\omega\right)\boldsymbol{\omega}_2 = 0\,.$$

The compatibility condition of this system leads to the dispersion equation

$$\xi^2 c_t^2\,(\chi + i) = i + \chi\,(1 + f_0\varepsilon)\,. \tag{24}$$

Of the two roots of Eq. (24), only the positive root is physically meaningful. In this case, the wave number q is determined by the formula

$$q = q' + iq'' = \frac{\omega}{c_t}\sqrt{1 + \frac{f_0\varepsilon\chi^2}{1+\chi^2} - i\,\frac{f_0\varepsilon\chi}{1+\chi}}\,,$$

where

$$q' = \frac{\omega}{\sqrt{2}\,c_t}\sqrt{\sqrt{\left(1 + \frac{f_0\varepsilon\chi^2}{1+\chi^2}\right)^2 + \left(\frac{f_0\varepsilon\chi}{1+\chi^2}\right)^2} + 1} \approx \frac{\omega}{c_t}\,;$$

$$q'' = \gamma_t = \frac{\omega}{\sqrt{2}\,c_t}\sqrt{\sqrt{\left(1 + \frac{f_0\varepsilon\chi^2}{1+\chi^2}\right)^2 + \left(\frac{f_0\varepsilon\chi}{1+\chi^2}\right)^2} - 1} \approx \frac{\omega}{\sqrt{t_2}\,c_t}\sqrt{\frac{f_0\varepsilon\chi^2}{1+\chi^2}}\,. \tag{25}$$

Thus, the third phase velocity of sound in snow coincides with the transverse velocity of sound propagation and does not depend on frequency and snow porosity; the coefficient of attenuation of transverse waves is $\gamma_t \sim \sqrt{f_0}$.

As follows from (25), the amplitude of the transverse wave decreases e-fold at a distance s determined as

$$s = \frac{1}{\gamma_t} = \frac{\sqrt{2}\,c_t}{\omega}\sqrt{\frac{1+\chi^2}{f_0\varepsilon\chi^2}}\,,$$

and the ratio of s to the wave length λ_t is

$$\frac{s}{\lambda_t} = \frac{1}{\sqrt{2}\,\pi} \sqrt{\frac{1+\chi^2}{f_0 \varepsilon \chi^2}} \; .$$

Since this ratio must be larger than unity, $\chi > (1 - 2\pi^2 f_0 \varepsilon)^{-1/2}$. Here $2\pi^2 f_0 \varepsilon \sim 10^{-2}$, consequently, $\chi > 1$ or

$$\frac{\nu_2}{k\omega} > 1 \; . \tag{26}$$

Condition (26) has quite a definite physical meaning: transverse elastic vibrations can propagate through the solid skeleton of snow only when snow has a rather large density (the coefficient χ is small). In this case, the parameter χ = 1 serves as a criterion that determines the possibility of propagation of transverse vibrations of the given frequency ω in the unbounded volume of snow.

When $\frac{\nu_2}{k\omega} < 1$, transverse waves cannot propagate in snow. In the operating range of frequencies of order 10^3 Hz, which is typical in practice, the coefficient of snow porosity must be larger than 10^{-4} cm^2.

When $\chi >> 1$, to which a larger snow density corresponds, from equality (25) we have

$$\gamma_t \approx \frac{\omega}{\sqrt{2}\, c_t} \sqrt{\frac{\rho_2}{\frac{1}{\rho} - \frac{1}{\rho_1}}} \sim \omega \; .$$

Thus, it follows from the studies that snow, as a porous medium, possesses three velocities of sound. Two of them are almost independent of the porous structure of snow and are determined only by elastic properties of the snow skeleton and snow density, whereas the pore velocity of sound, i.e., the velocity of propagation of elastic vibrations through air inclusions of snow, depends only on the dimensionless combination $\chi = \frac{\nu_2}{k\omega}$. This fact can be of importance for developing acoustic methods of determination of the pore velocity of sound in snow and the coefficient of permeability of snow χ, which plays a significant role in the processes of heat and mass transfer in the snow cover.

In conclusion, we note that the results given above can be fully applied to any porous medium consisting of solid particles of any shape and size tightly pressed to each other (e.g., grains of grass), the pores between these particles being filled by air or another gas that can freely pass through the solid skeleton of the medium in any direction.

NOTATION

c_2, velocity of sound in air, m/sec; c_l and c_t, longitudinal and transverse velocities of sound in the snow skeleton, m/sec; E, Young's modulus, Pa; f, coefficient of snow porosity; f_0, coefficient of porosity of snow in the absence of deformation; f', small additive to f; k, coefficient of snow permeability, m^2; K, K_1, and K_2, moduli of compressibility of snow, ice, and air, Pa; $m = c_l^2/c_2^2$, dimensionless constant of snow; q, wave number, m^{-1}; q' and q'', real and imaginary parts of the wave number q, m^{-1}; q_1, wave number for the solid skeleton of snow, m^{-1}; q_2, wave number of air in snow pores, m^{-1}; q_2' and q_2'', real and imaginary parts of the wave number q_2, m^{-1}; $\mathbf{u}$, small displacement of the element of the solid skeleton of snow, m; $\mathbf{v}_1$, mean macroscopic velocity of motion of particles of ice crystals, m/sec; $\mathbf{v}_2$, mean macroscopic velocity of motion of air in snow pores, m/sec; v_p, phase velocity of sound in the air confined in the snow pores, m/sec; v_g, group velocity (velocity of propagation of sound in the air confined in the snow pores or pore velocity of sound), m/sec; s, distance at which the amplitude of transverse waves in snow decreases e-fold; t, time, sec; x, coordinate, m; γ_2, coefficient of absorption of sound in the air confined in the snow pores, m^{-1}; γ_t, coefficient of absorption of transverse waves in snow, m^{-1}; δ, mean size of pores in snow, m; $\varepsilon = \frac{\rho_2}{\rho}$, small quantity; $\varepsilon' = \left(1 - f - \frac{K}{K_1}\right)\varepsilon$, small quantity; θ, relative bulk compression of an element of the structure of the snow skeleton; θ_0, amplitude of vibrations of relative bulk compression of an element of the structure of the snow skeleton; η_2, dynamic coefficient of air viscosity, kg/(m·sec); $\chi = \frac{\nu_2}{k\omega}$, dimensionless combination; λ, first Lamé coefficient, Pa; λ_2, length of a sound wave propagating through air inclusions in snow, m; λ_t, length of transverse waves in snow, m; μ, second Lamé coefficient, Pa; ν_2, kinematic coefficient of air viscosity, m^2/sec; $\xi = \frac{q}{\omega}$, $\xi_1 = \frac{q_1}{\omega}$, and $\xi_2 = \frac{q_2}{\omega}$, sec/m; ρ, snow density, kg/m^3; ρ_1, ice density, kg/m^3; ρ_2, air density, kg/m^3; $\rho_{2,0}$, density of air in snow pores in the absence of deformation, kg/m^3; ρ_2', small additive to ρ_2, g/m^3; σ, Poisson coefficient for snow; φ, relative bulk compression of air in snow pores; φ_0, amplitude of vibrations of relative bulk compression of pores in snow; ω, angular frequency of vibrations of the outer force, sec^{-1}; ω_1, angular velocity of an element of the solid skeleton of snow, sec^{-1}; ω_2, angular velocity of an element of air inclusions in snow, sec^{-1}; Δ, Laplace operator. Indices: p, phase; g, group; *l*, longitudinal; t, transverse.

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